THE JOURNAL OF TRANSPORT AND LAND USE http://jtlu.org
VOL. 6 NO. 1 [2013] pp. 89–100 doi: 10.5198/jtlu.v6i1.239

JTLU

The impact of transport, land and fiscal policy on housing and economic geography in a small, open growth model

Wei-Bin Zhang [a]
Ritsumeikan Asia Pacific University, Japan

Abstract: This study proposes a spatial model to examine dynamic interactions among growth, economic geography, the housing market, and public goods in a small, open economic growth model. We emphasize the impact of transport, land and fiscal policy on the spatial economy. The economy consists of the industrial sector, housing sector and local public goods. The model synthesizes the main ideas in the neoclassical growth theory, the Alonso urban model and the Muth housing model within the neoclassical open, small-growth framework. We solve the dynamics of the economic system and simulate the model to demonstrate dynamic interactions among economic growth, the housing market, residential distribution and public goods over time and space. Our simulation demonstrates, for instance, that as the tax rate on land income is increased, the total capital stocks and the stocks employed by the housing and public sectors are increased, the land devoted to local public goods falls and land rents and housing rents rise over space, and the consumption level of industrial goods and the total expenditures on public goods are increased. Our integrating model provides some new insights that cannot be obtained from the component models.

1 Introduction

The purpose of this study is to develop a spatial growth model to analyze urban configuration and economic growth within a compact framework. We synthesize the neoclassical economic growth theory and the standard land-use model in urban economics with public goods. It is well known that most neoclas¬sical models are extensions and generalizations of the pioneering works of Solow (1956) and Swan (1956). The standard neoclassical growth model initiated a new course of development of economic growth theory by using the neoclassical production function and neoclassical production theory. Although the early neoclassical growth models were developed for isolated economies without international trade, the one-sector growth model had been generalized to different cases for analyzing issues such as economic structural changes and international trade. There are mainly two analytical frameworks for analyzing trade in the neoclassical economic theory. The first one is the so-called Oniki-Uzawa trade model that studies interactions among growth and trade patterns between two economies. The second modeling framework analyzes economic growth of the so-called small, open economy for which the prices of tradable goods (such as physical capital and consumption goods) are fixed in the global market during the study period. The neoclassical growth theory has been generalized to analyze growth and capital accumulation for small, open economies. We refer to, for instance, Obstfeld and Rogoff (1996), Lane (2001), and Galí and Monacelli (2005), for the literature on economics of open economies. As far as trade with other countries is concerned, our model follows the neoclassical growth theory for small, open economies.

The spatial aspects of our model are based on typical growth models with economic geography. It is obvious that economic development and economic geography interact with each other over time. Economic growth, for instance, encourages demand for housing and affects prices and availability of land for housing. On the other hand, changes in the housing market will affect economic growth. For instance, as demand for housing is increased, demands for different services and goods and prices for different services and goods will be affected. Nevertheless, there are few economic models that deal with the interdependence of a micro-behavioral foundation, though the study of the economic growth with housing and economic geography has increasingly captured attention in urban economics and regional science. We refer the comprehensive surveys on the literature to Leung (2004), Henderson and Thisse (2004) and Capello and Nijkamp (2004) for the literature on the new economic geography. The necessity of explaining spatial evolution and growth with capital accumulation is pointed out by Baldwin and Martin (2004: 2675-6): "Many of the most popular economic geography models focus on labor.… These are unsuited to the study of growth." Capital accumulation is seldom modeled with land-use pattern and land markets in the literature of urban economics. Fujita and Thisse (2002:

[a] wbz1@apu.ac.jp

389) state the current situations of spatial economic growth as follows: "Clearly, space and time are intrinsically mixed in the process of economic development. However, the study of their interaction is a formidable task...Not surprisingly, therefore, the field is still in its infancy, and relevant contributions have been few." This study attempts to make a contribution to solving the long-standing puzzle of modeling economic growth with space by integrating the Alonso urban model and Muth housing model in urban economics with the neoclassical growth model for small, open economies. It should be noted that numerous contributions to urban economics have followed the equilibrium theory of urban land market pioneered by Alonso (1964). The earlier important contributions are carried out by, for instance, Muth (1969) and Solow (1973). The recent development, for instance, is referred to Lucas (1988), Henderson et al. (2001), Lucas and Rossi-Hansberg (2002) and Berliant et al. (2002). Most of the urban models neglect production aspects of urban dynamics.

Another contribution of this study is to introduce housing to spatial growth theory of small, open economies. A proper analysis of a housing market requires an explicit treatment of space. Housing is the largest component of nonhuman wealth for households, and housing services are a fundamental component of the household consumption. In the United States, for instance, real estate investment accounts for over 50 percent of total private investment and real estate assets represent just under 60 percent of the nation's wealth. Almost 70 percent of US real estate is residential (see DiPasquale and Wheaton, 1996). Housing has a set of intrinsic properties, which sets it apart from other goods. In the last three decades, many studies have been carried out to analyze durable housing in a spatial context (for instance, Muth, 1973; Anas, 1978; Hockman and Pines, 1980; Brueckner, 1981; Arnott, 1987; Brueckner and Pereira, 1994; Arnott et al., 1999; Braid, 2001). Nevertheless, as argued by Brito and Pereira (2002), the link between the housing market and long-term growth has been neglected in the literature. It is important to develop a growth model with housing market on microeconomic foundation.

This study also examines how the presence of exogenous growth mechanisms would change the analysis of the tax incidence of housing assets or the effects of different taxation of different assets. We refer the literature on studies of housing taxation to, for instance, Turnovsky and Okuyama (1994), Skinner (1996), Broadbent and Kremer (2001), and Burbidge and Cuff (2005). As in Hochman (1981) and Wijkander (1984), this paper is concerned with provision of local public goods in urban economies. We examine how the residential location pattern interacts with the provision of local public goods. We assume that only the government is responsible for the provision of public goods. The government chooses the values of a set of control measures according to some predetermined rules. The government provides public goods, minimizing the cost of public goods provision under these rules. The set of control measures at the government's disposal includes the amount of public goods in a fixed relation to the number of local residents, tax rates on the industrial sector's output, the housing sector's output, the land rent income, the wage income, and the wealth income. This paper studies the impact of transportation conditions, land size and government policies on economic growth and economic geography. We are concerned with the residential land-use pattern and determination of goods production, capital accumulation, local public goods provision, housing rents, and land rents over time and space. The model is a one-dimensional model of residential location with a central business center (CBD). We use the linear monocentric city as this allows us to explicitly analyze the model. This paper is based on a spatial growth model of an isolated state by Zhang (2010) but is different from Zhang in that this paper is concerned with the dynamics of a small, open economy. The paper is organized as follows. Section 2 defines the basic model. Section 3 shows how we solve the dynamics with economic geography. Section 4 examines effects of changes in some parameters on the dynamics of economic growth and geography. Section 5 concludes the study.

2 The Model

The model is a combination of the basic features of three key models, the Solow growth model, the Alonso urban model, and the Muth housing model, in neoclassical growth theory and urban economics. As far as urban structures are concerned, we follow the standard residential land-use model. All residents in the economy work in the CBD. People travel only between their homes and the CBD. Travel is equally costly in terms of time or/and money in all directions. An individual may reside at only one location. The spatial characteristic of any location that directly matters is the distance from the city center and the local public goods. The economy has a fixed population, We neglect international migration, but trade in goods and services is free. The economy consists of households, an industrial sector, the housing sector, and the government. The government taxes the industrial sector, the housing sector, wage income, income from owning wealth, and land rent income. The government uses all tax income to provide local public goods.

The system is geographically linear and consists of two parts: the CBD and the residential area. The economy consists of a finite strip of land extending from a fixed CBD with constant unit width. The households occupy the residential area.

We assume that the CBD is located at the left-side end of the linear territory. As we will get the same conclusions if we locate the CBD at the center of the linear system, the specified urban configuration will not affect our discussion. The economic geography is illustrated in Figure 1.

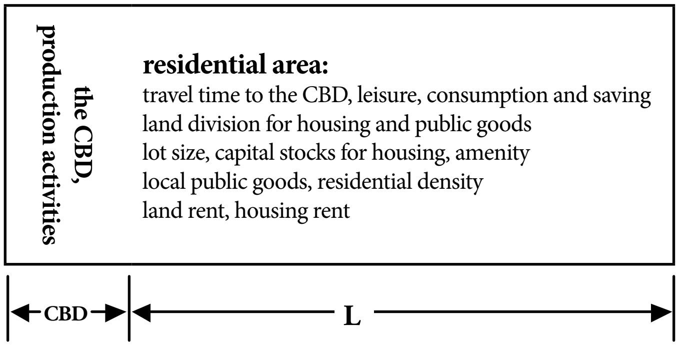

Figure 1: The spatial configuration of the isolated state

We assume that the total labor force is fully employed by the industrial sector. The industrial product can be either invested or consumed. Housing is supplied with a combination of capital and land. We select industrial goods to serve as numeraire. As we assume that the travel time of workers to the city is dependent on the travel distance, land rent for housing should be spatially different. We assume that industrial production is carried out by combination of capital, $K_i(t)$, and labor force in the form of

$$F=A_iK_i^{\alpha}N^{\beta},\ \alpha+\beta=1,\ \alpha,\ \beta>0.$$

where F is the output of the industrial sector. Markets are competitive; thus labor and capital earn their marginal products and firms earn zero profits. The rate of interest, r^* is determined in the global market (and thus is fixed for the small economy) and wage rate, $w(t)$ is determined in the domestic labor market. Hence, for any individual firm, r^* and $w(t)$ are given at each point of time. The production sector chooses the two variables, $K_i(t)$ and N to maximize its profit. The marginal conditions are given by

$$r^*+\delta_k=\frac{\alpha\tau_i F}{K_i},\ w=\frac{\beta\tau_i F}{N}$$

where $\bar{\tau}_i \equiv 1-\tau_i$, τ_i is the tax rate on the industrial product, and δ_k is the depreciation rate of physical capital. From the marginal conditions, we solve $K_i(t)$ and $w(t)$ as functions of r^* as follows

$$K_i=\left(\frac{\alpha\bar{\tau}_i A_i}{r^*+\delta_k}\right)^{1/\beta}N,\ w=\frac{\beta\bar{\tau}_i A_i}{N\alpha} \quad (1)$$

We see that K_i, F, and w are constant for fixed r^*.

We assume that all housing is residential housing. The housing industry supplies housing services by combining land and capital. We use $n(\omega, t)$ and $L_h(\omega, t)$ to stand for the number of residents and the lot size of the household at location ω. Let $c_h(\omega, t)$ denote housing service received by the household at location ω, $0 \leq \omega \leq L$ We specify the housing service production function as follows

$$c_h(\omega, t) = A_h k_h^{\alpha_h}(\omega, t) L_h^{\beta_h}(\omega, t),\ \alpha_h+\beta_h=1,\ \alpha_h, \beta_h>0 \quad (2)$$

where $k_h(\omega, t)$ is the input level of capital per household at location ω. Let $R(\omega, t)$ and $R_h(\omega, t)$ stand for, respectively, the land rent and housing rent at location ω. The marginal conditions are given by

$$r^* + \delta_k = \frac{\bar{\tau}_h\alpha_h R_h c_h}{k_h},\ R = \frac{\bar{\tau}_h\alpha_h R_h c_h}{L_h},\ 0 \leq \omega \leq L, \quad (3)$$

in which $\tau_h = 1 - \tau_h$, and τ_h is the tax rate of housing product.

The land is divided between housing production and local public goods provision. In this study, we neglect issues, such as congestion, endogenous travel time, and tolls, related to transportation systems by assuming that transportation systems are predetermined and there is no congestion. The division of land at any particular location will affect the number of households living at that location as well as the level of local public goods provision. For the given levels of capital stocks for housing and local public goods, if more land were allocated to housing, there would be room for a larger number of households, whereas the width for public services would fall. Let $L_p(\omega, t)$ stand for the amount of land for public goods provision at location ω According to the definitions of L_p, L_n and n, we have

$$n(\omega, t)L_h(\omega, t) + L_p(\omega, t) = 1,\ 0 \leq \omega \leq L, \quad (4)$$

The amount of land used for housing and public goods adds up to the total amount available.

The total capital stocks employed by the housing sector, $K_h(t)$, is equal to the sum of the capital stocks for housing over space at any point in time. The relationship between $k_h(\omega, t)$ and $K_h(t)$ is thus given by

$$K_h(t) = \int_0^L n(\omega, t)k_h(\omega, t)d\omega. \quad (5)$$

We assume that the land is equally owned by the population. This implies that the revenue from land is equally shared among the population. The land market is assumed to be competitive.

The total land revenue, $R(t)$ is given by $R(t)=\int_0^L R(\omega, t)d\,\omega$. The income from land per household is given by $r(t)=R(t)/N$. Consumers make decisions on choice of lot size and consumption level of commodity as well as on how much to save. This study uses the approach to consumers' behavior proposed by Zhang in the early 1990s. The approach to household behavior in this study is discussed at length by Zhang (2008). Let $k(\omega, t)$ stand for the per capita wealth (excluding land) owned by the typical household in a location ω Let τ_k, τ_w and τ_L stand for, respectively, the tax rates on wealth income, wage income and land income. Introduce $\bar{\tau}_j \equiv 1 - \tau_j$, $j = k, w, L$. Each household at ω obtains income

$$y(\omega, t) = \bar{\tau}_k r^* k(\omega, t) + \bar{\tau}_w w + \bar{\tau}_L \bar{r}(t), \quad 0 \le \omega \le L \quad (6)$$

from the interest payment, $\tau_k r^* k$, the wage payment, $\tau_w w$, and the land rent income, $\tau_L r$. We call $y(\omega, t)$ the current income in the sense that it comes from consumers' wages and current earnings from ownership of wealth. The total value of the wealth that a consumer at location can sell to purchase goods and to save is equal to $p_i(t)k(\omega, t)$, with $p_i(t)$=1 at any t. Here, we assume that selling and buying wealth can be conducted instantaneously without any transaction cost. The disposable income is then equal to

$$\hat{y}(\omega, t) = y(\omega, t) + k(\omega, t) \quad (7)$$

The disposable income is used for saving and consumption. It should be noted that the value, $k(\omega, t)$, (i.e., $p_i(t)k(\omega, t)$), in the above equation is a flow variable. Under the assumption that selling wealth can be conducted instantaneously without any transaction cost, we may consider k as the amount of income that the consumer at ω obtains at time t by selling all of his or her wealth. Hence, at time t the consumer has the total amount of income equaling $\hat{y}$ to distribute between consuming and saving.

At each point in time, a consumer at location ω distributes the total available budget among leisure time, $T_h(\omega)$, housing, $c_h(\omega, t)$ saving, $s(\omega, t)$, and consumption of industrial goods, $c(\omega, t)$. Here, we assume that leisure is only dependent on the residential location as work time is fixed and equal for each household, regardless of residential location. After work time is decided, households decide the time distribution between leisure and travel to work. As we assume that the travel time from the CBD to the residential location is only related to the distance and neglect any other effects such on technological change, infrastructure improvement, and congestion on the travel time from the CBD to the residential area, the leisure time, which is equal to the fixed total time minus the travel time, is only related to location. Let T_0 and $\Gamma(\omega)$, respectively, stand for the total available time for travel and leisure and the time spent on traveling between the residence and the CBD. We have $T_h(\omega)=T_0 - \Gamma(\omega)$. The budget constraint is given by

$$R_h(\omega, t)c_h(\omega, t) + c(\omega, t) + s(\omega, t) = \hat{y}(\omega. t). \quad (8)$$

Equation (8) means that consumption and saving exhaust consumers' disposable personal income.

Location choice is related to the quality of physical environmental attributes as open space and noise pollution as well as social environmental quality. We assume that utility level, $U(\omega, t)$, of the household at location ω is dependent on $T_h(\omega)$, $c_h(\omega, t)$, $s(\omega, t)$ and $c(\omega, t)$ as follows

$$U(\omega, t) = \theta(\omega, t)\, T_h^{\sigma}(\omega)c^{\xi_0}(\omega, t)c_h^{\eta_0}(\omega, t)s^{\lambda_0}(\omega, t),\ \sigma, \xi_0, \eta_0, \lambda_0 > 0, \quad (9)$$

in which σ, ξ_0, η_0, and λ_0 are a typical person's elasticity of utility with regard to leisure time, industrial goods, housing, and saving at ω We call σ, ξ_0, η_0, and λ_0 propensities to use leisure time, to consume goods, to consume housing, and to hold wealth, respectively. As argued by Glaeser et al. (2001), consumption amenities have increasingly played a more important role in urban formation. In this study, we incorporate amenity into the consumer location decision by assuming that amenity is a function of residential density. We consider that residential densities may have positive or negative agglomeration effects. We specify the amenity, $\theta(\omega, t)$ at ω as follows

$$\theta(\omega, t) = \theta_1 E(\omega, t)n^{\mu_i}(\omega, t),\ \theta_1 > 0 \quad (10)$$

where θ_1 and μ_i are constant. The function, $\theta(\omega,t)$ implies that the amenity level at location ω is related to the residential density at the location and the local public services, $E(\omega,t)$.

We assume that there is only one type of public good: pure public goods. It is also assumed that the only people who receive benefits from public goods are residents at the location of their provision—people living at the distance from which these goods are provided. For simplicity, we assume that the level of public goods is related to the land input and capital input. We specify the local service supply function as follows

$$E(\omega,t) = A_p k_p^{\alpha_{p0}}(\omega,t) L_p^{\beta_{p0}}(\omega,t),\ \alpha_{p0}, \beta_{p0} \geq 0 \tag{11}$$

where $k_p(\omega,t)$ is the level of capital input at location ω and α_{p_0}, β_{p_0}, and A_p are positive parameters. We neglect labor input in public service. Let $y_p(\omega,t)$ stand for the public expenditure at location ω. The government will maximize the supply of public goods subject to its local budget constraint. Hence, the government problem is defined by

$Max\ E(\omega, t)$

$s.t. : (r^* + \delta_k)k_p(\omega, t) + R(\omega, t)Lp(\omega, t) = yp(\omega, t).$

The optimal solution of the government's behavior is described by

$$(r^* + \delta_k)k_p(w, t) = \alpha_p y_p(\omega, t),\ R(\omega, t)L_p(w,t) = \beta_p y_p(\omega,t), \tag{12}$$

where

$$\alpha_p \equiv \frac{\alpha_{p^0}}{\alpha_{p^0}+\beta_{p^0}},\ \beta_p \equiv \frac{\beta_{p^0}}{\alpha_{p^0}+\beta_{p^0}}$$

To model how the government determines the distribution of $y_p(\omega, t)$ we assume that the government supplies the public goods at location ω at time t as follows

$$E(\omega, t) = \theta_0 n^\mu(\omega, t),\ \theta_0, \mu_a > 0. \tag{13}$$

The supply of public services at the location is positively related to the distribution of households. For convenience of analysis, we assume θ_0 to be independent of location and time. This is a strict requirement. For instance, the government may encourage the population distribution to be flatter by providing more services to remote places, that is, $d\theta_0 / d\omega > 0$ By (10) and (13), we have $\theta(\omega, t) = \bar{\theta} n^\mu(\omega, t)$ where $\mu_i + \mu_\alpha$ *and* $\bar{\theta} = \theta_0\theta_1$.

The total capital stocks employed by the public service sector is given by

$$K_p(t)=\int_0^L k_p(\omega, t)d\omega. \tag{14}$$

The total expenditure of the public service sector is financed by the tax incomes. The total expenditure of the public sector, Yp(t) is given by $Y_p(t)=\int_0^L y_p(\omega, t)d\omega$ As the public sector is financed by government tax income, we have

$$Y_p(t)=\tau_i F(t)+\tau_k r^* K(t)+\tau_w Nw(T)+\tau_L R(t)+\tau_h\int_0^L R_h(\omega, t)c_h(\omega, t)n(\omega, t)d\omega, \tag{15}$$

where the left-hand side is the total expenditure of the public service sector, $\tau_i F(t)$ is the tax income from the industrial sector, $\tau_k r^* K(t)$ is the tax on the households' income from wealth, $\tau_w Nw(t)$ is the tax on the households' wages, $\tau_L \bar{R}(t)$ is the tax on the land, and $\tau_h \int_0^L R_h c_h n d\omega$ is the tax income from the housing sector.

As the population is homogeneous and people can change their residential location freely without any transaction costs and time delay, it is reasonable to assume that all households obtain the same level of utility at any point in time. The conditions that households get the same level of utility at any location at each point in time is represented by

$$U(\omega_1,t) = U(\omega_2,t),\ 0 \leq \omega_1, \omega_2 \leq L. \tag{16}$$

This is a strict requirement as time and money are required for any individual household to change dwelling location. Equation (16) determines economic geography over the residential area.

Maximizing $U(\omega, t)$ subject to the budget constraint (8) yields

$$c(\omega, t) = \xi\hat{y}(\omega, t),\ c_h(\omega, t) = \frac{\eta\hat{y}(\omega, t)}{R_h(\omega, t)},\ s(\omega, t) = \lambda\hat{y}(\omega, t), \tag{17}$$

in which

$$\xi \equiv \frac{\xi_0}{\xi_0 + \eta_0 + \lambda_0},\ \eta \equiv \frac{\eta_0}{\xi_0 + \eta_0 + \lambda_0},\ \lambda \equiv \frac{\lambda_0}{\xi_0 + \eta_0 + \lambda_0}$$

The above equations mean that housing consumption, consumption of goods, and savings are positively proportional to the available income.

According to the definition of $s(\omega, t)$ the capital accumulation for the household at location ω is given by

$$k(\omega, t) = s(\omega, t) - k(\omega, t),\ 0 \leq \omega \leq L. \tag{18}$$

As the state is isolated, the total population is distributed over the whole urban area. The population constraint is given by

$$\int_0^L n(\omega, t)d\omega = N \tag{19}$$

The total consumption, $C(t)$ is given by

$$\int_0^L n(\omega, t)c(\omega, t)d\omega = C(t) \tag{20}$$

The national production is equal to the national consumption and national net saving. The assumption that capital is fully employed is given by

$$K_i(t) + K_h(t) + K_p(t) = K(t), \tag{21}$$

where the total capital stocks employed by the production sectors is equal to the total wealth owned by all the households. That is

$$\int_0^L k(\omega, t)n(\omega, t)d\omega = K(t) \tag{22}$$

We have thus built the dynamic growth model with endogenous spatial distribution of wealth, consumption and population, capital accumulation and residential location. Next we examine dynamic properties of the system.

3 The dynamics and equilibrium

Before examining the dynamic properties of the system, we show that the dynamics can be expressed by a single differential equation with the variable, $K(t)$ In the rest of the paper, we omit ω or/and t in the expressions, wherever without causing confusion. First, we define the total disposable income as $Y(t) \equiv \int_0^L \hat{y}(\omega, t)n(\omega, t)\, d\omega$ We also introduce a few parameters for the convenience of representation

$$\tilde{\beta}_0 = \lambda\bar{\tau}_w N w + \lambda\bar{\tau}_L \tilde{\alpha}_0, \tilde{\beta} \equiv 1 - \lambda - \lambda\bar{\tau}_k r^* - \lambda\bar{\tau}_L \tilde{\alpha}, \tilde{\alpha}_0 \equiv \frac{\beta_p \tilde{r} b_p}{\alpha_p} + \frac{\beta_h \tilde{r} b_h}{\alpha_h}$$

$$\tilde{\alpha} \equiv (\beta_p + \beta_h)\tilde{r}, \tilde{r} \equiv r^* + \delta_k, w_0 \equiv \tau_i F + \tau_w Nw, \tilde{r}_h \equiv \frac{\tau_L \beta_h \tilde{r}}{\alpha_h} + \frac{\tilde{r}\tau_h}{\bar{\tau}_h \alpha_h}$$

$$a_h \equiv \frac{1 + \bar{\tau}_k r^* + \bar{\tau}_0 \tilde{r}_0 \tau_k r^*}{(1 - \tau_0 \tilde{r}_0)\tilde{r}_h}, b_h \equiv \frac{\bar{\tau}_w w N}{(1 - \tau_0 \tilde{r}_0)\tilde{r}_h}, a_p \equiv (\tau_k r^* + \tilde{r}_h a_h)\tilde{r}_0, \tilde{r}_0 \equiv \frac{\alpha_p}{r(1 - \tau_L \beta_p)}$$

$$b_p \equiv \tilde{r}_0 w_0 + \tilde{r}_0 \tilde{r}_h b_h, \tilde{r}_h \equiv \left(\frac{1}{\bar{\tau}_h \eta}\right) - \bar{\tau}_L \beta_h \frac{\tilde{r}}{\alpha_h}, \bar{\tau}_0 \equiv \frac{\bar{\tau}_L \beta_p \tilde{r}}{\alpha_p}, \alpha_0 \equiv \left(\frac{\theta_0}{\alpha^{\alpha_{p0}} \beta^{\beta_{p0}} E_p}\right)^{1/(\alpha p0 + \beta p0)}$$

3.1 Lemma 1

Assume that the initial wealth distribution, $k(\omega, 0)$ is independent of location, that is
$k(\omega_1, 0) = k(\omega_2, 0), 0 \leq \omega_1, \omega_2 \leq L$.
Then, the dynamics of the (non-land) wealth owned by the country, $K(t)$ is given by

$$K(t) = K(0)e^{-\tilde{\beta}t} + \frac{\tilde{\beta}_0}{\tilde{\beta}} \tag{23}$$

At any point in time, the location-independent variables are determined as a unique function of $K(t)$ by the following procedure: $K_p = a_p K + b_p$ *and* $K_h = a_h K + b_h \rightarrow \tilde{K}$ by (21) $\rightarrow Y_p = \tilde{r} K_p / \alpha_p \rightarrow \bar{R} = \tilde{\alpha}_0 + \tilde{\alpha} K \rightarrow \bar{r} = \bar{R}/N \rightarrow Y = \tilde{r} K_h / \tau_h \alpha_h \eta \rightarrow C = \xi \hat{Y}$ and $S = \lambda \hat{Y} \rightarrow k = K/N \rightarrow K_i$ and w by (1) $\rightarrow F(K_i, N) \rightarrow \hat{y} = \hat{Y}/N \rightarrow k_h + K_h/N \rightarrow c = C/N$ and $s = S/N$. Moreover, if we further assume that $\alpha_p = 1/2$ *and* $\mu_a = \alpha_{p_0}$ then we determine all location-dependent variables as functions of K by the following procedure:

$$n(0) = \frac{NT_h^{\beta_0}(0)}{\int_0^L T_h^{\beta_0}(\omega)}, n(\omega) = n(0)\left(\frac{T_h(\omega)}{T_h(0)}\right)^{\beta_0}, \beta_0 \equiv 2\sigma/(\eta_0 - 2\mu)$$

$$L_h(\omega) = \frac{1}{n(\omega)} \frac{\Lambda_r^2 + 4 - \Lambda_r}{2}, \quad R = \frac{(r^* + \delta_k)\beta_h k_h}{\alpha_h L_h}, \quad \Lambda_r(K) \equiv \alpha_0 \beta_p \left(\frac{\alpha_h}{\beta_h k_h}\right)^{\alpha_p}$$

$$L_h(\omega) = \frac{1}{n(\omega)} \frac{\Lambda_r^2 + 4 - \Lambda_r}{2}, \quad y_p = \alpha_0 (r^* + \delta_k)^{\alpha_p} R^{\beta_p} n^{\mu_a/(\alpha_{p0} + \beta_{p0})}$$

where $\alpha_0 \equiv \left(\theta_0 / \alpha_p^{\alpha_{p0}} \beta_p^{\beta_{p0}} E_p\right)^{1/(\alpha_{p0} + \beta_{p0})}$ $\rightarrow$ $k_p(\omega, t)$ by (12) $\rightarrow$ $E(\omega, t)$ by (13) $\rightarrow$ $\theta(\omega, t)$ by (10) $\rightarrow$ $c_h(\omega, t)$ by (2) $\rightarrow$ $R_h(\omega, t)$ by (3) $\rightarrow$ $U(\omega, t)$ by (9).

The lengthy proof of the lemma is available on request from the author. Lemma 1 means that for a given rate of interest, we can determine all the variables over time and space, such as the industrial sector's output, housing product, capital distribution between the two sectors, rate of interest, wage rate, income from land ownership, income and wealth distribution over space, residential distribution, total transportation time spent in the system, leisure time distribution and total leisure time in the economy, land rent, housing rent, housing consumption, consumption of industrial goods, production of public services, tax incomes from the different sources, and land distribution between housing production and public services.

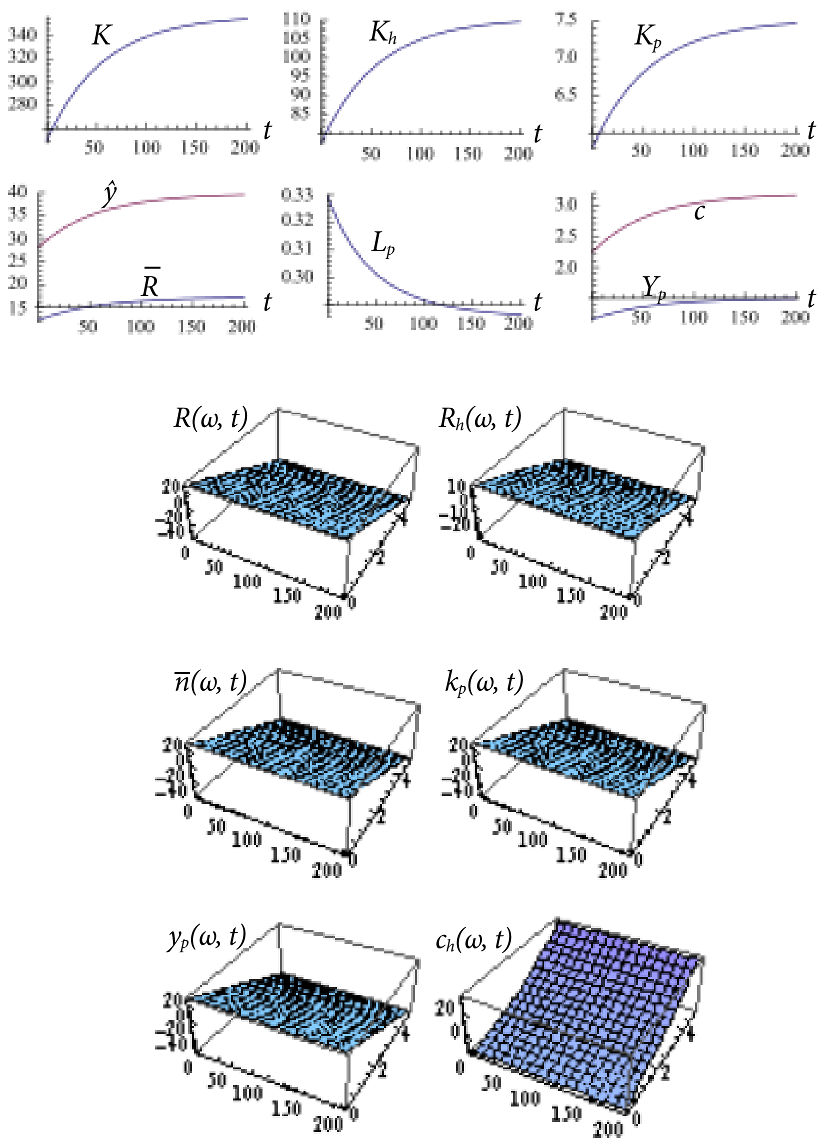

Figure 2: The motion of the spatial economy

4 The spatial dynamics with the Cobb-Douglas production function

As the expressions are too complicated, it is difficult to explicitly interpret the results. We illustrate the results by simulation. We specify the travel time function as follows: $\Gamma(\omega) = \upsilon\omega$, in which $1/\upsilon$ is the travel speed. In this study, we assume travel speed is constant. We neglect possible factors, such as change in transportation technology or congestion, which may affect travel speed. For illustration, we now specify values of the parameters as follows:

$$r^* = 0.05,\ A_i = 1.2,\ N = 10,\ L = 5,\ \alpha = 0.3,\ A_h = 0.9,\ \alpha_h = 0.4,\ \eta_0 = 0.07,\ \xi_0 = 0.08,\ \lambda_0 = 0.85,\ \sigma = 0.2,\ \mu_i = -0.4,\ \mu_a = 0.35,\ \alpha_{p0} = \beta_{p0} = 0.35,\ \theta_0 = 0.5,\ A_p = 0.4,\ T_0 = 1,\ \upsilon = 0.05,\ \tau_i = 0.02,\ \tau_w = 0.02,\ \tau_k = 0.02,\ \tau_L = 0.03,\ \tau_h = 0.01,\ \delta_k = 0.05. \qquad (24)$$

The rate of interest is fixed at 5 percent. The population is fixed at 10 units, and the urban size is fixed at 5 units. The productivity of the industrial sector and the housing sector are specified respectively with 1.2 and 0.9. We specify α with 0.3 partly because some empirical studies with the Cobb-Douglas production function uses this value. The propensities to consume goods and consume housing are respectively specified at 0.08 and 0.07. The propensity to save is 0.85. The propensity

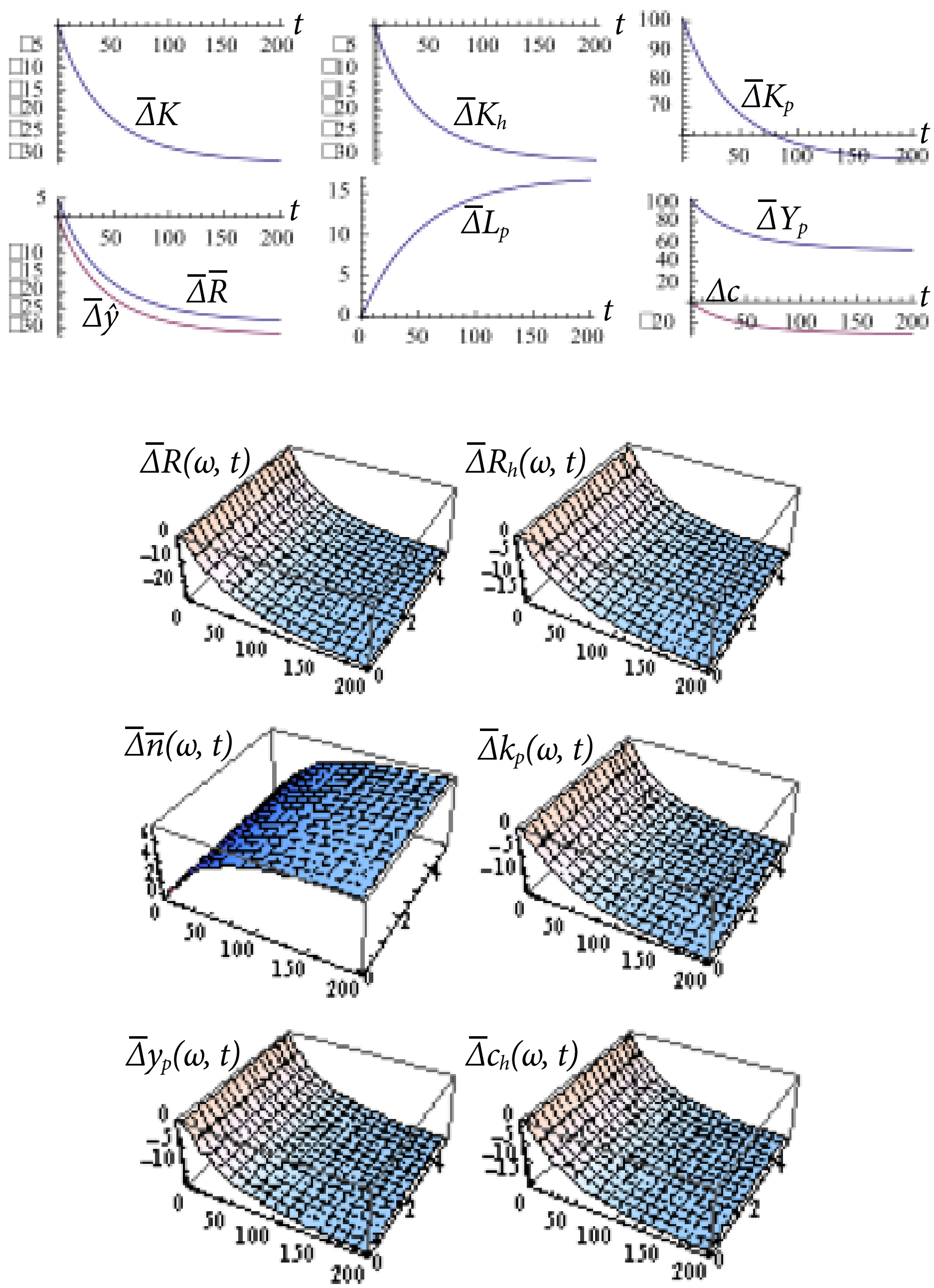

Figure 3: A rise in the tax rate on land rent income

to use leisure is specified at 0.2. The amenity parameter, μ_i, is negative. This implies that households prefer to live in an area with low residential distribution. The total available time is fixed at one unit and $v = 0.05$ means that total travel time from the CBD to the other end of the system will use up 25 percent of the total available time. The depreciation rate is specified at 0.05. Tax rates on industrial output, wage rate, and income from wealth are 2 percent. Tax rates on the land income and housing output are, respectively, 3 percent and 1 percent. As we have explicitly provided the procedure to determine all the variables, the specified values will not affect our simulation.

Following Lemma 1 under (24), we calculate the dynamics of the small, open economy. As $\beta = 0.018$, by Lemma 1 we conclude that the dynamic system is stable. The system has a unique stable equilibrium point static state. The equilibrium value of $K(t)$ is given by $\beta_0 / \beta = 357.2$. It is straightforward to calculate the equilibrium values of all the other variables. We calculate the values of the three variables, which are independent of time and space, as

$$K_i = 6.06,\ w = 0.708,\ F = 10.32.$$

We simulate the motion of the system with the initial condition, $K(0) = 250$. The motion of the system is plotted in Figure 2. We see that the variable, $K(t)$, at its initial state is much lower than its long equilibrium value, which is about 65.9. Figure 3 illustrates how the whole system approaches its equilibrium over time. The total capital stocks and the capital stocks employed by the two sectors rise over time. The total income from the land is increased as the economy grows and the land devoted to public goods falls. The total output, available income, and consumption all rise over time. The total expenditure on local public goods provision is expanded over time. We also illustrate how the space-dependent variables change over time. Both land rents and housing rents rise over time at any location. The residential density falls at each location (in association with the decrease of land devoted to providing public goods). Housing consumption per household, the government's expenditures on local public goods, and the capital stocks employed at any location all increase. Although the land devoted to local public goods provision falls over time, public services are improved over time because more capital stocks are employed by the government. It should be noted that the residential density, $n(\omega)$, is given by $n(\omega)\ 1 / L_h(\omega)$. Hence, for the two variables, $n(\omega)$ and $L_h(\omega)$ it is sufficient to be concerned with one variable. In the rest of the paper, we are concerned with residential density.

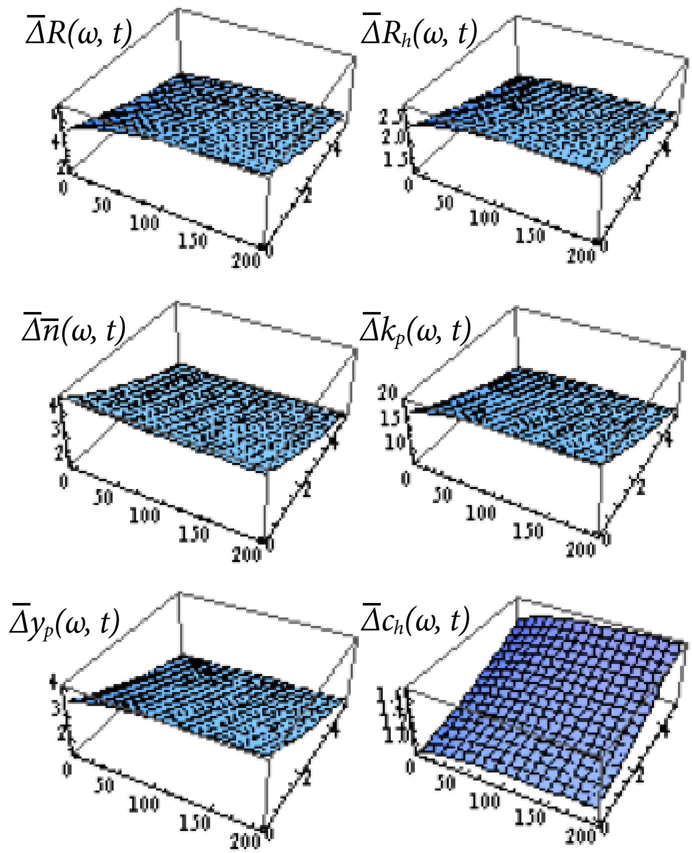

Figure 4: A reduction in the travel speed

5 Comparative dynamics analysis with regard to some parameters

First, we are concerned with possible effects of change in the tax rate on land income. We now consider a case that the tax rate on land income, τ_L is increased from 3 percent to 12 percent and the other parameters are kept the same as in (24). We introduce a symbol, $\Delta x(t)$, to stand for the change rate of the variable $x(t)$ in percentage due to changes in the parameter value. The change has no impact on K_i, w and F. We plot the effects on the other variables in Figure 3. As the tax rate on land income is increased, the total capital stocks and stocks employed by housing are reduced, but the capital employed by the public sector is increased. The income from taxing the land is increased, and the land devoted to local public goods rises. The consumption level of industrial goods is reduced and total expenditures on public goods are increased. The land rent and housing rent fall over time and space. The residential density rises at each location. This occurs as the land for the public goods is reduced and thus land for housing is increased. The capital stocks for and expenditures on local public services are

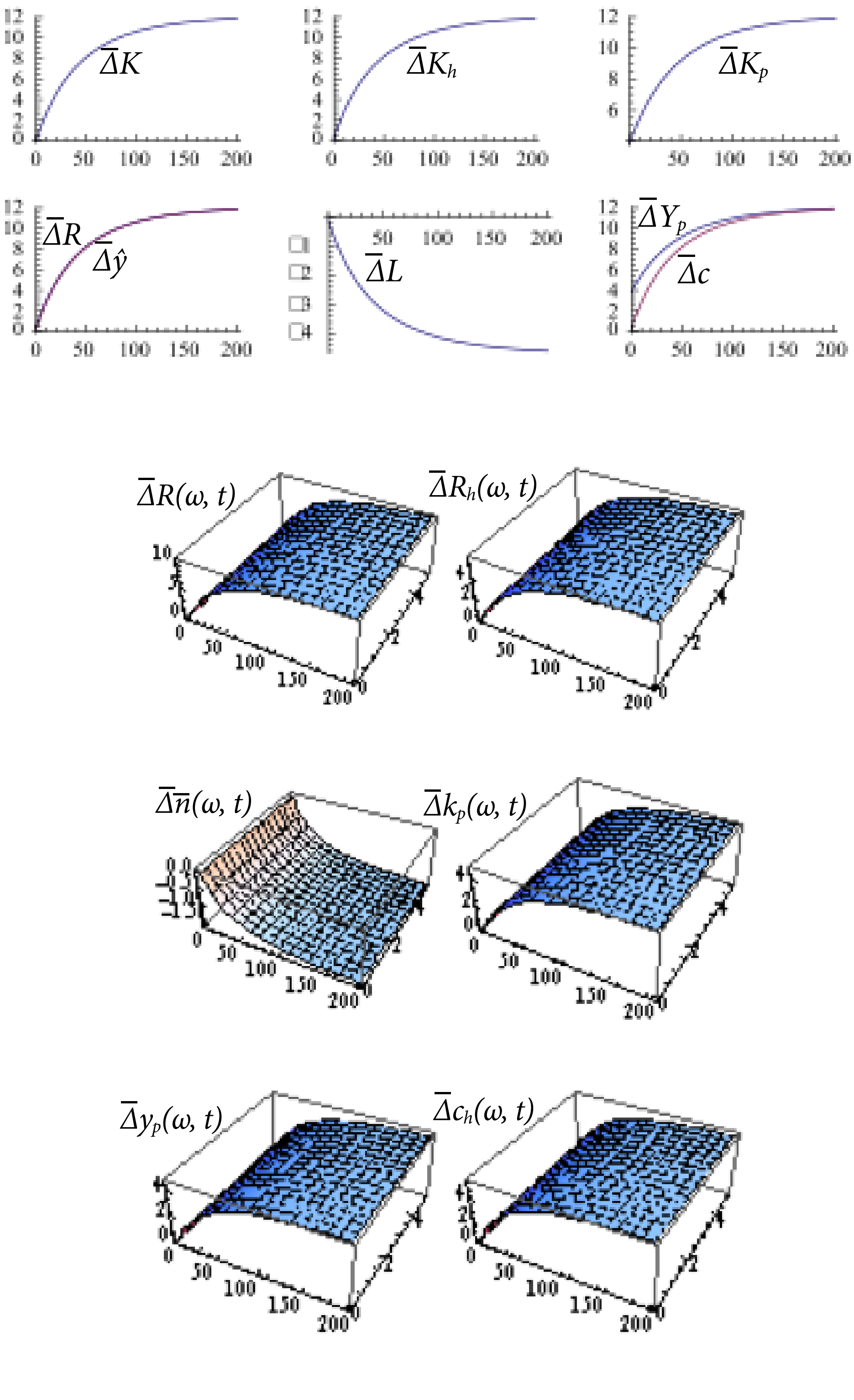

Figure 5: An improvement in the industrial sector's productivity

reduced at each location. Housing consumption per household falls.

We now study what will happen if the transportation conditions are changed in the following way: $\upsilon : 0.05 \rightarrow 0.08$. This implies that travel speed is reduced, for instance, due to deterioration of road conditions or congestion. The location-independent variables are not affected by the parameter. The effects on the location-dependent variables are plotted in Figure 4. We see that as travel speed is reduced, land and housing rents are increased near the CBD but reduced far away from the CBD. The residential density becomes higher near the CBD but lower far away from the CBD.

We now increase the productivity of the industrial sector, A_i from 1.3 to 1.3 The changes in the values of the three variables, which are independent of time and space, are given by

$$\bar{\Delta}K_i = \bar{\Delta}w = \bar{\Delta}F = 12.11.$$

The effects on the other variables are plotted in Figure 5. The total capital stocks employed by the economy rise over time. The capital stocks of the other two sectors are increased. Income from land rises over time. The land devoted to local public goods falls. The consumption level is increased. The total expenditure on the public rises. Land rents and housing rents rise over time and space. The residential density falls.

6 Concluding remarks

This study proposed a spatial model with local public goods by synthesizing the main ideas in the three key models in neoclassical growth theory and urban economics. The economic growth with economic geography has a unique long-run stable. We solve the dynamics of the economic system and simulate the model to demonstrate dynamic interactions among economic growth, housing market, residential distribution and public goods over time and space. Our simulation demonstrates, for instance, that as the tax rate on land income is increased, total capital stocks and stocks employed by the housing and public sectors are increased, the land devoted to local public goods falls and land rents and housing rents rise over space, the consumption level of industrial goods and total expenditures on public goods are increased. Our model can be extended in different ways. For instance, we may extend the model to take into account different land uses of the urban area. More realistic representations of housing market dynamics and transportation systems with congestion can also be taken into account. Taxation should be endogenous variables and there are different kinds of public goods. In many urban models, the CBD is no more fixed than assumed in this study. As shown in Fujita and Thisse (2002: Chap. 6), various urban forms can be explained due to communication externalities.

References

Alonso, W. 1964. *Location and land use*. Cambridge, MA: Harvard University Press.

Anas, A. 1978. Dynamics of urban residential growth. *Journal of Urban Economics*, 5: 66–87. http://dx.doi.org/10.1016/0094-1190(78)90037-2

Arnott, R. J. 1987. Economic theory and housing. In *Handbook of regional and urban economics*, Volume II, edited by E. S. Mills. Amsterdam: Elsevier North-Holland.

Arnott, R., R. Braid, R. Davidson, and D. Pines. 1999. A general equilibrium spatial model of housing quality and quantity. *Regional Science and Urban Economics*, 29: 283–316. http://dx.doi.org/10.1016/S0166-0462(98)00035-0

Baldwin, R. E. and P. Martin. 2004. Agglomeration and regional growth. In *Handbook of regional and urban economics*, Volume IV: Cities and Geography. Amsterdam: Elsevier North-Holland.

Berliant, M., S. K. Peng, and P. Wang. 2002. Production externalities and urban configuration. *Journal of Economic Theory*, 104: 275–303. http://dx.doi.org/10.1006/jeth.2001.2847

Braid, R. M. 2001. Spatial growth and redevelopment with perfect foresight and durable housing. *Journal of Urban Economics*, 49: 425-52. http://dx.doi.org/10.1006/juec.2000.2199

Brito, P. M. B., and A. M. Pereira, 2002. Housing and endogenous long-term growth. *Journal of Urban Economics*, 51: 246–71. http://dx.doi.org/10.1006/juec.2001.2244

Broadbent, B., and M. Kremer. 2001. Does favorable tax-treatment of housing reduce non-housing investment? *Journal of Public Economics*, 81: 369-91. http://dx.doi.org/10.1016/S0047-2727(00)00125-0

Brueckner, J. K. 1981. A dynamic model of housing production. *Journal of Urban Economics*, 10: 1–14. http://dx.doi.org/10.1016/0094-1190(81)90019-X

Brueckner, J. K., and A. M. Pereira. 1994. Housing ownership and the business cycle. *Journal of Housing Economics*, 3: 165–85. http://dx.doi.org/10.1006/jhec.1994.1007

Burbidge, J., and K. Cuff. 2005. Capital tax competition and returns to scale. *Regional Science and Urban Economics*, 35: 353-73. http://dx.doi.org/10.1016/j.regsciurbeco.2004.05.003

Capello, R., and P. Nijkamp. 2004. *Urban dynamics and growth: Advances in urban economics*. Amsterdam: Elsevier.

DiPasquale, D., and W. C. Wheaton. 1996. *Urban economics and real estate markets*. Englewood Cliffs, NJ: Prentice Hall.

Fujita, M., and J. F. Thisse. 2002. *Economics of agglomeration: Cities, industrial location, and regional growth*. Cambridge, MA: Cambridge University Press.

Gali, J., and T. Monacelli. 2005. Monetary policy and exchange rate volatility in a small open economy. *Review of Economic Studies*, 72: 707–34. http://dx.doi.org/10.1111/j.1467-937X.2005.00349.x

Glaeser, E. L., J. Kolko, and A. Saiz. 2001. Consumer city. *Journal of Economic Geography*, 1: 27–50. http://dx.doi.org/10.1093/jeg/1.1.27

Henderson, J. V., Z. Schalizi, and A. J. Venables. 2001. Geography and development. *Journal of Economic Geography*, 1: 81–105. http://dx.doi.org/10.1093/jeg/1.1.81

Henderson, J. V., and J. F. Thisse. 2004. *Handbook of regional and urban economics*. Amsterdam: Elsevier.

Hochman, O. 1981. Land rents, optimal taxation and local fiscal independence in an economy with local public goods. *Journal of Public Economics*, 15: 59–85. http://dx.doi.org/10.1016/0047-2727(81)90053-0

Hochman, O., and D. Pines. 1980. Costs of adjustment and the spatial pattern of a growing open city. *Econometrica*, 50: 1371–89. http://www.jstor.org/stable/1913387

Lane, P. R. 2001. The new open economy macroeconomics: a survey. *Journal of International Economics*, 54: 235–66. http://dx.doi.org/10.1016/S0022-1996(00)00073-8

Leung, C. 2004. Macroeconomics and housing: A review of the literature. *Journal of Housing Economics*, 13: 249–67. http://dx.doi.org/10.1016/j.jhe.2004.09.002

Lucas, R. E. 1988. On the mechanics of economic development. *Journal of Monetary Economics*, 22: 3–42. http://dx.doi.org/10.1016/0304-3932(88)90168-7

Lucas, R. E., and E. Rossi-Hansberg. 2002. On the internal structure of cities. *Econometrica*, 70: 1445–76. http://www.jstor.org/stable/3082004

Muth, R. F. 1969. *Cities and housing*. Chicago: University of Chicago Press.

Muth, R. F. 1973. A vintage model of the housing stock. *Papers of the Regional Science Association*, 30: 141–156. http://dx.doi.org/10.1007/BF01941811

Obstfeld, M., and K. Rogoff. 1996. *Foundations of international macroeconomics*. Cambridge, MA: MIT Press.

Skinner, J. 1996. The dynamic efficiency cost of not taxing housing. *Journal of Public Economics*, 59: 397-417. http://dx.doi.org/10.1016/0047-2727(95)01509-4

Solow, R. 1956. A contribution to the theory of growth. *Quarterly Journal of Economics*, 70: 65–94. http://www.jstor.org/stable/1884513

Solow, R. M. 1973. On equilibrium models of urban location. In *Essays in modern economics*, edited by M. Parkin, 2-16. London: Longman.

Swan, T. W. 1956. Economic growth and capital accumulation. *Economic Record*, 32: 334–61. http://dx.doi.org/10.1111/j.1475-4932.1956.tb00434.x

Turnovsky, S., and T. Okuyama. 1994. Taxes, housing, and capital accumulation in a two-sector growing economy. *Journal of Public Economics*, 53: 245–67. http://dx.doi.org/10.1016/0047-2727(94)90023-X

Wijkander, H. 1984. Provision of public goods in congested cities. *Journal of Public Economics*, 25: 127–41. http://dx.doi.org/10.1016/0047-2727(94)90023-X

Zhang, W. B. 2008. *International trade theory: Capital, knowledge, economic structure, money and prices over time and space*. Berlin: Springer.

Zhang, W. B. 2010. Economic growth with space and fiscal policies with housing and public goods. *Journal of Economic Studies* (in process).